

CREDIT RISK: CONCENTRATION RISK

Migration Analysis

method to measure loan concentration risk by tracking credit ratings of firms in particular sectors
rating classes for unusual declines

Example

		Risk rating classes at end of yr				
		1	2	3	Default	
risk rating classes at beg of yr	1	0.85	0.10	0.04	0.01	1.00
	2	0.12	0.83	0.03	0.02	1.00
	3	0.03	0.13	0.80	0.04	1.00

=> BNK would limit loans in classes 2, 3
or ask for higher returns

cells represent transition probabilities
the Mx is called Load migration matrix

Concentration limits

external limits set on the maximum loan size that can be made to an individual borrower/sector

Example

p = portfolio
c = percentual of portfolio 'p' allocated to borrower/sector 'c'
= concentration limit
lr = loss rate for sector 'c' = 0.40
ll = loss limit on portfolio allowed by sector 'c' = 0.10 p

$$c * lr = ll \quad \Rightarrow \quad c = ll / lr \Rightarrow \quad \boxed{c = 0.25 p}$$

Modern Portfolio Theory

Loan	w	R	s	s ²	w*R	w ² * s ²
1	0.40	0.10	0.980	0.960	0.0400	0.1537
2	0.60	0.12	0.857	0.734	0.0720	0.2644
	1.00					

corr12 = -0.84
s12 = -0.705 = corr12 * s1 * s2

Rp = 0.1120
= $\sum R_j * w_j$

s2p = 0.079
= $\sum s_j^2 * w_j^2 + \sum \sum w_i * w_j * \text{corrij} * s_i * s_j$

sp = 0.282

sp < [s1, s2]

MPT application: KMV Portfolio Manager Model

Loan	W	s	f	AIS	EDF	LGD	E(L)	s(d)	R	s	W * R	W^2 * s^2
1	0.60	0.050	0.020	0.070	0.030	0.250	0.008	0.171	0.0625	0.04	0.0375	0.0007
2	0.40	0.045	0.015	0.060	0.020	0.200	0.004	0.140	0.0560	0.03	0.0224	0.0001
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	1.00											

$$\text{corr12} = -0.250$$

$$s12 = 0.000 = \text{corr12} * s1 * s2$$

R = return of the loan
= AIS - E(L)

AIS = annual all-in-spread = annual fees + spread
= s + f
s = spread
f = annual fees

E(L) = expected loss of Loan
= EDF * LGD

EDF = expected default frequency

LGD = loss given default

s = risk of the loan
= s(d) * LGD

s(d) = std dev of default rate of borrower
= (EDF (1 - EDF)^ 0.5 * LGD (ass: bin distr)

Rp = 0.0599 expected return
s2p = 0.0006 variance
sp = 0.0252 risk of the loan = unexpected loss of the loan

Partial MPT application: Loan Volume-Based Model

Loan Categories	X n	X 1	X 2	(X1-Xn)^2	X2-Xn
1	0.45	0.65	0.10	0.04	0.1225
2	0.30	0.20	0.25	0.01	0.0025
3	0.15	0.10	0.55	0.0025	0.16
4	0.10	0.05	0.10	0.0025	0
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	1.00	1.00	1.00		

X n = national asset allocation proportion
X = BNK asset allocation proportion

s = std dev of BNK from national allocation
= (∑ (X - X n)^2 / N)^0.5

N = number of loan categories

s1 = 0.117
s2 = 0.267

BNK 2 deviates more significantly from National bnchmrk than BNK 1 because of heavy concentration on category 3

Partial MPT application: Loss Ratio-Based Model

the model is based on the estimation of the systematic loan loss risk of a particular sector

Example

from historical data: $X_a = 0.003 + 0.75 X_L$
 $X_b = 0.005 + 1.25 X_L$

X_a = loan loss rate in sector a of Financial Institution

X_b = loan loss rate in sector b of Financial Institution

X_L = loan loss rate of loan portfolio of Financial Institution

if $X_L = 0.15$

then $X_a = 0.116$

$X_b = 0.193$

=> to protect the FI, I should reduce the exposure on sector b